

پاسخنامه سوالات شبه نهایی ریاضی 3 دوازدهم تجربی

1-الف) نادرست ب) نادرست ج) درست د) درست

2-الف) 6 ب) $[-4, 12)$ ج) $\frac{2}{3}$

3- الف) $D_f = (-\infty, 2]$, $D_g = R - \{0\}$

$$D_{f \circ g} = \left\{ x \mid x \in R - \{0\}, \frac{x-1}{x} \leq 2 \right\} \rightarrow \frac{x-1}{x} - 2 \leq 0 \rightarrow \frac{-x-1}{x} \leq 0$$

x		-1	0
$-x-1$	+	-	-
x	-	-	+
$\frac{-x-1}{x}$	-	+	-

$$D_{f \circ g} = (-\infty, -1] \cup (0, +\infty)$$

(ب) $(g \circ f)^{-1}(3) = f^{-1} \circ g^{-1}(3) = f^{-1}(g^{-1}(3))$

$$g^{-1}(3) \rightarrow 3 = \sqrt{x+7} \rightarrow 9 = x+7 \rightarrow x = 2 \rightarrow g^{-1}(3) = 2$$

$$f^{-1}(2) \rightarrow \frac{1}{5}x - 3 = 2 \rightarrow \frac{1}{5}x = 5 \rightarrow x = 25 \rightarrow f^{-1}(g^{-1}(3)) = 25$$

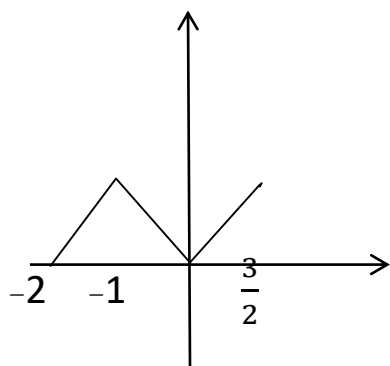
4- الف) $\max = |a| + c = |-1| + \sqrt{3} = 1 + \sqrt{3}$

$$\min = -|a| + c = -|-1| + \sqrt{3} = -1 + \sqrt{3}$$

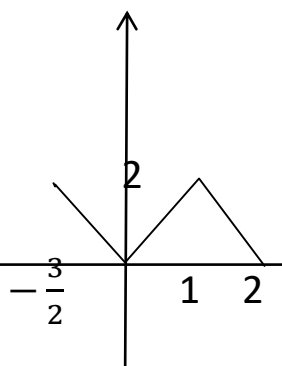
(ب) $4 \sin x \cos x = \sqrt{3} \rightarrow 2(2 \sin x \cos x) = \sqrt{3} \rightarrow \sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3}$

$$\begin{cases} 2x = 2k\pi + \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6} \\ 2x = (2k+1)\pi - \frac{\pi}{3} \rightarrow x = \frac{(2k+1)\pi}{2} - \frac{\pi}{6} \end{cases}$$

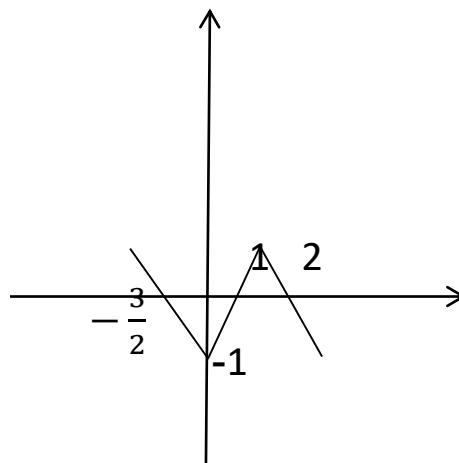
-5



$$y = f(2x)$$



$$y = f(-2x)$$



$$y = f(-2x) - 1$$

$$\text{الف) } \lim_{x \rightarrow 1^+} \frac{[x]}{1 - |x|} = \frac{[1^+]}{1 - 1^+} = \frac{1}{0^-} = -\infty$$

$$\text{ب) } \lim_{x \rightarrow -1} \frac{2x + \sqrt{3-x}}{x^2 + x} \times \frac{2x - \sqrt{3-x}}{2x - \sqrt{3-x}} = \lim_{x \rightarrow -1} \frac{4x^2 - 3 + x}{x(x+1)(2x - \sqrt{3-x})} =$$

$$\lim_{x \rightarrow -1} \frac{(x-1)(4x-3)}{x(x+1)(2x - \sqrt{3-x})} = -\frac{7}{4}$$

$$\text{ج) } \lim_{x \rightarrow +\infty} \left(\frac{4x+1}{x^2+1} - \frac{2x^2+1}{3x^2-2} \right) = \lim_{x \rightarrow +\infty} \frac{4x}{x^2} - \lim_{x \rightarrow +\infty} \frac{2x^2}{3x^2} = 0 - \frac{2}{3} = -\frac{2}{3}$$

$$y' = 5 \left(\frac{x-1}{2x-1} \right)^4 \times \frac{1(2x-1) - 2(x)}{(2x-1)^2}$$

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الف)

$$\text{ب) } y' = \frac{5}{2^2 \sqrt{5x+2}} (-x+3) - 1(\sqrt{5x+2})$$

$$v' = 80 \left(0 - \frac{1}{100} \right) \left(1 - \frac{t}{100} \right) = -\frac{80}{100} \left(1 - \frac{t}{100} \right)$$

-8 آهنگ تغییر لحظه‌ای

$$[0, 100] \text{ در بازه } \text{آهنگ متوسط} = \frac{40(1-1)^2 - 40(1-0)^2}{100-0} = -\frac{40}{100}$$

$$1 - \frac{t}{100} = \frac{1}{2} \rightarrow t = 50$$

$$(1, 1) \rightarrow 1 = 2(1)^3 + a(1) + b \rightarrow a + b = -1$$

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$$y' = 6x^2 + a \rightarrow f'(1) = 0 \rightarrow 6 + a = 0 \rightarrow a = -6, b = 5$$

$$y = 2x^3 - 6x + 5, \quad y' = 6x^2 - 6 = 0 \rightarrow x = \pm 1$$

$$f(-1) = 9 \rightarrow (-1, 9) \text{ ماکزیمم نسبی}$$

$$y = 3x^2 - 3x + 1 \rightarrow y' = 6x - 3 = 0 \rightarrow x = \frac{1}{2} \quad f\left(\frac{1}{2}\right) = \frac{1}{4} \quad \text{10-مینیمم مطلق}$$

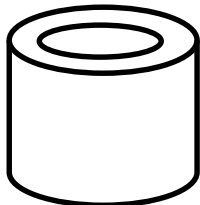
$$x = 0 \rightarrow f(0) = 1, \quad x = 4 \rightarrow f(4) = 37 \quad \text{ماکزیمم مطلق}$$

$$2a = 12 \rightarrow a = -11$$

$$6, FF' = \sqrt{(1-1)^2 + (-5-3)^2} = 8, \quad 2c = 8 \rightarrow c = 4$$

$$a^2 = b^2 + c^2 \rightarrow 36 = b^2 + 16 \rightarrow b^2 = 20 \rightarrow b = 2\sqrt{5} \rightarrow BB' = 4\sqrt{5}$$

$$-12 \quad v = \pi(3)^2 \times 4 = 36\pi \quad \text{استوانه بزرگ}$$



$$v = \pi(1)^2 \times 4 = 4\pi \quad \text{استوانه داخل}$$

$$v = 36\pi - 4\pi = 32\pi \quad \text{حجم بین دو استوانه}$$

$$o\left(-\frac{a}{2}, -\frac{b}{2}\right) = (1, -2), \quad r = \frac{1}{2}\sqrt{a^2 + b^2 - 4c} = \frac{1}{2}\sqrt{4 + 16 - (-4)} = 3$$

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$$o(-1, 2), r = \frac{1}{2}\sqrt{4 + 16 + 36} = \frac{1}{2}\sqrt{56} = \sqrt{14}$$

$$oo = \sqrt{(-1 - 1)^2 + (2 - (-2))^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

$$|r - r'| < oo < r + r \rightarrow \sqrt{14} - 3 < 2\sqrt{5} < \sqrt{14} + 3 \quad \text{دو دایره متقاطعند}$$

$$x^2 - 4 = 0 \rightarrow x = \pm 2 \quad -14$$

x		-2	2
$x^2 - 4$		+	-
		-	+

$$y = \begin{cases} x^2 - 4 & x < -2, x > 2 \\ -x^2 + 4 & -2 \leq x \leq 2 \end{cases} \quad y' = \begin{cases} 2x & x < -2, x > 2 \\ -2x & -2 < x < 2 \end{cases}$$

$$f'_+(2) = 4 \neq f'_-(2) = -4$$

$$x - y = 10 \rightarrow y = x - 10 \quad -15$$

$$f(x) = xy = x(x - 10) = x^2 - 10x \rightarrow f'(x) = 2x - 10 = 0 \rightarrow x = 5$$

$$y = 5 - 10 = -5$$

$$p(\text{فرزند سالم}) = p(\text{پسر}) \times p(\text{پسر} | \text{سالم}) + p(\text{دختر}) \times p(\text{سالم} | \text{دختر}) \quad -16$$

$$p(\text{فرزند سالم}) = \frac{1}{2} \times (1 - 0.08) + \frac{1}{2} \times (1 - 0.03) = 0.046 + 0.485 = 0.945$$

پیروز و سربلند باشید.